COLLEGE OF ENGINEERING

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Blacksburg, Virginia 24061

Nonlinear Wave Choked Inlets

Semiannual Progress Report

Period: October 1, 1978 - April 30, 1979

The two basic changes that were done to the quasi-one dimensional flow program, where the replacement of the Runge-Kutta subroutine with a subroutine using a modified divided difference form of the Adams Pece Method and the replacement of the matrix inversion routine with a pseudo-inverse routine. The following system of first-order differential equations is solved in the program:

$A \frac{dy}{dx} = B$, where A is a n x n matrix,

y and B are n-component column vectors.

At the throat of the duct, if the flow is approaching choked conditions then $\det A \to 0$ and the system is singular. The pseudo-inverse routine determines, towards the solution of the linear system $AZ = B$, the singular value decomposition, $A = USV^T$. Householder biodiagonalization and a variant of the QR algorithm are used. The output from this routine is S, V, and G, where $G = U^T B$; S is diagonal and V,U are orthogonal; S also determines whether A is singular. If one or more of the elements of S is zero then A is singular. In the subroutine, if $S_{ii} \leq 10^{-14}$ then S_{ii}^{-1} is set equal to zero. The program then solves the following system

$$SV^T \frac{dy}{dx} = U^T B = G$$

$$V^T \frac{dy}{dx} = S^{-1} G = S_{ii}^{-1} G_i$$

If $\frac{G_i}{S_{ii}} \geq 100$ the program stops since this indicates that y' is becoming too large for the integration routine to handle. Therefore incompatible reflection coefficients (c_R's) have been chosen. If the above check is passed, then y' is integrated. Solving for y' we have

$$y' = VS^{-1}G$$

Now these values are passed to the integration routine. Unlike the Runge-Kutta routine which would integrate to the end of the duct even if incorrect c_R's were input, the present routine will not pass through the throat. The first cases considered were for one reflection coefficient where

$$\rho_{11}^{L}(0) = c_R\rho_{11}^{R}(0), \text{ and } c_R\rho_{11}^{R}(0)$$

were specified. The initial conditions at x = 0 for the higher harmonics were set equal to zero.

By varying c_R the following plot was obtained showing the region of correct c_R's. This region was determined using two harmonics. Later cases using four and ten harmonics indicated that the region did not shrink by much.

The following examples compare the original Runge-Kutta version of the program with the modified version with the Adams-Pece and pseudo-inverse subroutines. The input to both runs is identical except for the c_R's. For the Runge-Kutta case, c_R = (0.,0.) and the results are for two cases, two and four harmonics. It is seen that as the throat is approached detA → 0 and there is a substantial growth in the amplitudes of the harmonics. In the case of four harmonics there are very large jumps in the amplitudes near the throat. This indicates that incorrect c_R's were chosen.

The example using the modified program was run with $c_R = (-.5, -3.3)$. As is seen there are no large oscillations in the harmonics near the throat. This implies that the integration to the right end is valid. In the former cases, the Runge-Kutta routine could not detect the singularity and was integrating across it. Therefore, the values obtained by the Runge-Kutta method in the vicinity of the throat to the right end are incorrect. If this occurs in the new version, it will terminate at the throat where

$$RU_n = U_{1n} - \frac{c_0}{\rho_0} \rho_{1n}$$

$$RT_n = T_{1n} - \frac{(\gamma-1)T_0}{\rho_0} \rho_{1n}$$

$\omega = 1$

$M_0 = -.4$

$\rho_I = .005$

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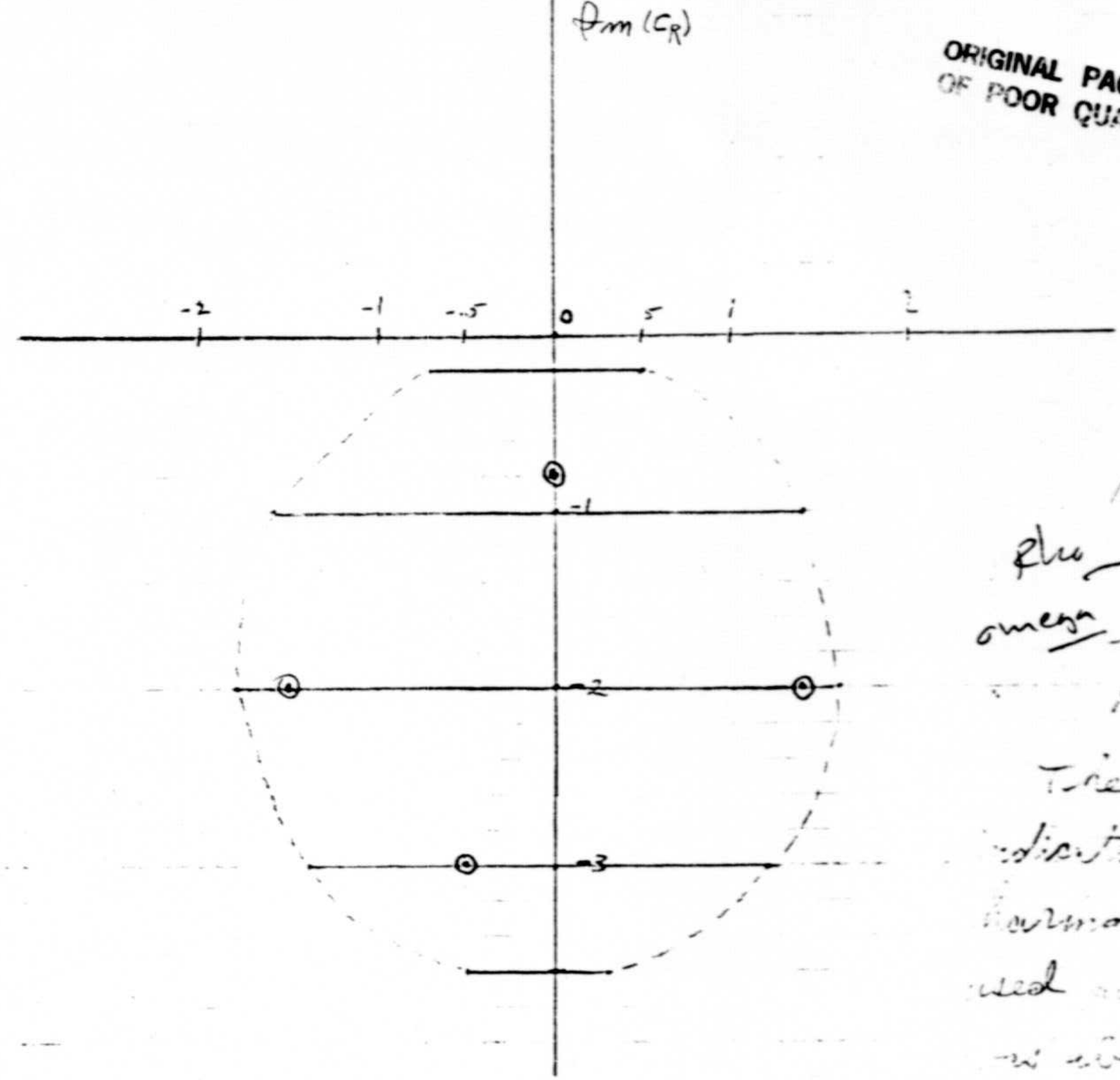

Re(C_R)

$M_0 = $ [illegible]

Rho $C_I = $ [illegible]

omega $=$ [illegible]

$NH = 2$

The red [illegible] indicate [illegible] harmonics were used and the pro[illegible] was able to integrate through the duct.

NUMBER OF HARMONICS CONSIDERED= 4

W= 1.000 XMCO=-0.4000 XF= 2.000 A2= 0.100 DD= 1.000
CR= -0.5000 -3.3000 RI= 0.0050 NX= 80 NX1= 80 XMU= 0.0

X	X/L	DET	MACH NO.	FIRST HARMON.	SECOND HARMON.	THIRD HARMON.	FOURTH HARMON.
0.02500	0.00302	0.28D 07	-0.40050	0.98584	0.00043	0.00001	0.00000
0.05000	0.00604	0.27D 07	-0.40120	0.97329	0.00086	0.00002	0.00000
0.07500	0.00906	0.24D 07	-0.40271	0.96233	0.00127	0.00003	0.00000
0.10000	0.01208	0.21D 07	-0.40484	0.95292	0.00169	0.00004	0.00000
0.12500	0.01510	0.18D 07	-0.40759	0.94506	0.00212	0.00005	0.00000
0.15000	0.01812	0.14D 07	-0.41096	0.93875	0.00257	0.00006	0.00000
0.17500	0.02115	0.84D 06	-0.41496	0.93397	0.00304	0.00007	0.00000
0.20000	0.02417	0.84D 06	-0.41960	0.93074	0.00353	0.00008	0.00000
0.22500	0.02719	0.42D 06	-0.42489	0.92907	0.00405	0.00009	0.00000
0.25000	0.03021	0.42D 06	-0.43084	0.92896	0.00461	0.00010	0.00000
0.27500	0.03323	0.18D 06	-0.43747	0.93045	0.00522	0.00011	0.00001
0.30000	0.03625	0.18D 06	-0.44477	0.93356	0.00587	0.00012	0.00000
0.32500	0.03927	0.18D 06	-0.45277	0.93832	0.00658	0.00013	0.00000
0.35000	0.04229	0.66D 05	-0.46147	0.94477	0.00735	0.00013	0.00000
0.37500	0.04531	0.66D 05	-0.47089	0.95297	0.00819	0.00014	0.00000
0.40000	0.04833	0.21D 05	-0.48105	0.96296	0.00910	0.00016	0.00000
0.42500	0.05135	0.21D 05	-0.49195	0.97481	0.01012	0.00017	0.00000
0.45000	0.05437	0.56D 04	-0.50360	0.98858	0.01123	0.00019	0.00000
0.47500	0.05739	0.56D 04	-0.51602	1.00437	0.01247	0.00022	0.00000
0.50000	0.06042	0.13D 04	-0.52922	1.02226	0.01386	0.00026	0.00000
0.52500	0.06344	0.13D 04	-0.54319	1.04235	0.01542	0.00031	0.00001
0.55000	0.06646	0.25D 03	-0.55794	1.06476	0.01720	0.00039	0.00001
0.57500	0.06948	0.25D 03	-0.57347	1.08964	0.01923	0.00049	0.00002
0.60000	0.07250	0.25D 03	-0.58978	1.11712	0.02159	0.00062	0.00003
0.62500	0.07552	0.41D 02	-0.60685	1.14739	0.02435	0.00080	0.00004
0.65000	0.07854	0.41D 02	-0.62466	1.18061	0.02763	0.00105	0.00006
0.67500	0.08156	0.54D 01	-0.64320	1.21701	0.03160	0.00139	0.00009
0.70000	0.08458	0.54D 01	-0.66241	1.25677	0.03647	0.00188	0.00014
0.72500	0.08760	0.56D 00	-0.68225	1.30017	0.04261	0.00260	0.00022
0.75000	0.09062	0.56D 00	-0.70264	1.34727	0.05039	0.00364	0.00036
0.77500	0.09364	0.16D 00	-0.72350	1.39821	0.06059	0.00524	0.00061
0.80000	0.09666	0.43D-01	-0.74468	1.45283	0.07429	0.00780	0.00107
0.82500	0.09969	0.11D-01	-0.76601	1.51050	0.09318	0.01205	0.00200
0.85000	0.10271	0.24D-02	-0.78723	1.56956	0.11994	0.01944	0.00397
0.87500	0.10573	0.24D-02	-0.80796	1.62627	0.15876	0.03264	0.00820
0.90000	0.10875	0.51D-03	-0.82766	1.67240	0.21612	0.05744	0.01821
0.92500	0.11177	0.11D-03	-0.84543	1.69084	0.30003	0.10434	0.04223
0.95000	0.11479	0.39D-04	-0.85990	1.65003	0.41410	0.18783	0.09715
0.97500	0.11781	0.16D-04	-0.86921	1.51108	0.53816	0.30658	0.19818
1.00000	0.12083	0.14D-04	-0.87154	1.26678	0.61554	0.40886	0.31557
1.02500	0.12385	0.33D-04	-0.86658	0.97396	0.61547	0.44431	0.38866
1.05000	0.12687	0.10D-03	-0.85597	0.70977	0.57149	0.43148	0.40815
1.07500	0.12989	0.36D-03	-0.84145	0.52148	0.51425	0.40002	0.39524
1.10000	0.13291	0.13D-02	-0.82424	0.43263	0.45698	0.36409	0.36760
1.12500	0.13593	0.46D-02	-0.80527	0.43328	0.40471	0.32929	0.33573

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1.15000	0.13896	0.14D-01	-0.78520	0.46129	0.35889	0.29771	0.30451
1.17500	0.14198	0.70D-01	-0.76453	0.54224	0.31942	0.26990	0.27600
1.20000	0.14500	0.19D 00	-0.74364	0.60122	0.28561	0.24577	0.25105
1.22500	0.14802	0.31D 00	-0.72279	0.65366	0.25665	0.22497	0.22933
1.25000	0.15104	0.13D 01	-0.70219	0.69893	0.23178	0.20704	0.21054
1.27500	0.15406	0.50D 01	-0.68199	0.73769	0.21034	0.19156	0.19443
1.30000	0.15708	0.12D 02	-0.66229	0.77093	0.19181	0.17814	0.18046
1.32500	0.16010	0.26D 02	-0.64319	0.79962	0.17574	0.16646	0.16834
1.35000	0.16312	0.56D 02	-0.62474	0.82463	0.16179	0.15623	0.15776
1.37500	0.16614	0.24D 03	-0.60698	0.84669	0.14967	0.14725	0.14858
1.40000	0.16916	0.47D 03	-0.58996	0.86641	0.13916	0.13931	0.14044
1.42500	0.17218	0.91D 03	-0.57369	0.88428	0.13006	0.13226	0.13333
1.45000	0.17520	0.17D 04	-0.55818	0.90073	0.12223	0.12599	0.12705
1.47500	0.17822	0.31D 04	-0.54344	0.91611	0.11553	0.12039	0.12145
1.50000	0.18125	0.96D 04	-0.52949	0.93068	0.10984	0.11537	0.11649
1.52500	0.18427	0.96D 04	-0.51630	0.94471	0.10508	0.11087	0.11209
1.55000	0.18729	0.24D 05	-0.50389	0.95840	0.10113	0.10682	0.10818
1.57500	0.19031	0.56D 05	-0.49224	0.97192	0.09794	0.10318	0.10471
1.60000	0.19333	0.56D 05	-0.48134	0.98544	0.09540	0.09990	0.10161
1.62500	0.19635	0.12D 06	-0.47119	0.99907	0.09347	0.09695	0.09887
1.65000	0.19937	0.23D 06	-0.46176	1.01296	0.09205	0.09430	0.09643
1.67500	0.20239	0.23D 06	-0.45306	1.02720	0.09110	0.09193	0.09428
1.70000	0.20541	0.42D 06	-0.44506	1.04190	0.09055	0.08982	0.09238
1.72500	0.20843	0.42D 06	-0.43776	1.05715	0.09035	0.08793	0.09072
1.75000	0.21145	0.71D 06	-0.43114	1.07303	0.09044	0.08627	0.08929
1.77500	0.21447	0.11D 07	-0.42519	1.08964	0.09079	0.08482	0.08809
1.80000	0.21749	0.11D 07	-0.41990	1.10705	0.09135	0.08356	0.08710
1.82500	0.22052	0.16D 07	-0.41526	1.12533	0.09210	0.08246	0.08617
1.85000	0.22354	0.20D 07	-0.41126	1.14458	0.09299	0.08157	0.08550
1.87500	0.22656	0.20D 07	-0.40790	1.16486	0.09400	0.08082	0.08500
1.90000	0.22958	0.25D 07	-0.40516	1.18626	0.09511	0.08022	0.08466
1.92500	0.23260	0.25D 07	-0.40303	1.20885	0.09630	0.07977	0.08448
1.95000	0.23562	0.28D 07	-0.40153	1.23273	0.09755	0.07946	0.08446
1.97500	0.23864	0.28D 07	-0.40063	1.25799	0.09884	0.07926	0.08462
2.00000	0.24166	0.28D 07	-0.40035	1.28470	0.10016	0.07922	0.08495

Original Version

NUMBER OF HARMONICS CONSIDERED= 2

W= 1.000 XMCU=-0.4000 XF= 2.000 A2= 0.100 DD= 1.000
CR= 0.0 0.0 RI= 0.0050 NX= 80 NXI= 80 XMU= 0.0

X	X/L	DFT	MACH NO.	FIRST HARMON.	SECOND HARMON.	THIRD HARMON.	FOURTH HARMON.
0.02500	0.00083	0.40D 04	-0.40030	1.00104	0.00051		
0.05000	0.00167	0.38D 04	-0.40121	1.00416	0.00103		
0.07500	0.00250	0.37D 04	-0.40273	1.00937	0.00155		
0.10000	0.00333	0.35D 04	-0.40487	1.01673	0.00209		
0.12500	0.00417	0.32D 04	-0.40762	1.02626	0.00265		
0.15000	0.00500	0.29D 04	-0.41099	1.03804	0.00323		
0.17500	0.00583	0.26D 04	-0.41499	1.05215	0.00384		
0.20000	0.00667	0.22D 04	-0.41963	1.06867	0.00450		
0.22500	0.00750	0.19D 04	-0.42492	1.08773	0.00519		
0.25000	0.00833	0.16D 04	-0.43087	1.10945	0.00595		
0.27500	0.00917	0.13D 04	-0.43749	1.13402	0.00677		
0.30000	0.01000	0.11D 04	-0.44479	1.16160	0.00767		
0.32500	0.01083	0.88D 03	-0.45278	1.19242	0.00867		
0.35000	0.01167	0.69D 03	-0.46148	1.22673	0.00978		
0.37500	0.01250	0.53D 03	-0.47090	1.26484	0.01104		
0.40000	0.01333	0.40D 03	-0.48104	1.30710	0.01245		
0.42500	0.01417	0.30D 03	-0.49193	1.35390	0.01406		
0.45000	0.01500	0.22D 03	-0.50357	1.40573	0.01590		
0.47500	0.01583	0.16D 03	-0.51597	1.46316	0.01813		
0.50000	0.01667	0.11D 03	-0.52914	1.52684	0.02069		
0.52500	0.01750	0.78D 02	-0.54309	1.59758	0.02373		
0.55000	0.01833	0.55D 02	-0.55781	1.67633	0.02736		
0.57500	0.01917	0.35D 02	-0.57330	1.76422	0.03177		
0.60000	0.02000	0.23D 02	-0.58957	1.86264	0.03717		
0.62500	0.02083	0.15D 02	-0.60659	1.97328	0.04387		
0.65000	0.02167	0.94D 01	-0.62434	2.09821	0.05229		
0.67500	0.02250	0.58D 01	-0.64230	2.23999	0.06304		
0.70000	0.02333	0.34D 01	-0.66192	2.40185	0.07697		
0.72500	0.02417	0.20D 01	-0.68166	2.58773	0.09534		
0.75000	0.02500	0.11D 01	-0.70193	2.80277	0.12002		
0.77500	0.02583	0.63D 00	-0.72264	3.05340	0.15389		
0.80000	0.02667	0.34D 00	-0.74367	3.34783	0.20141		
0.82500	0.02750	0.17D 00	-0.76485	3.69650	0.26976		
0.85000	0.02833	0.80D-01	-0.78596	4.11260	0.37065		
0.87500	0.02917	0.41D-01	-0.80671	4.61251	0.52374		
0.90000	0.03000	0.18D-01	-0.82675	5.21614	0.76268		
0.92500	0.03083	0.77D-02	-0.84565	5.94843	1.14671		
0.95000	0.03167	0.28D-02	-0.86313	6.85639	1.78933		
0.97500	0.03250	0.24D-03	-0.88424	8.09791	3.33410		
1.00000	0.03333	-0.38D-03	-0.88838	6.01471	2.43900		
1.02500	0.03417	0.30D-02	-0.86438	5.64190	2.66151		
1.05000	0.03500	0.55D-02	-0.85385	5.01716	2.64332		
1.07500	0.03583	0.11D-01	-0.83912	4.38383	2.49934		
1.10000	0.03667	0.23D-01	-0.82174	3.83049	2.30710		
1.12500	0.03750	0.48D-01	-0.80268	3.36909	2.10735		
1.15000	0.03833	0.98D-01	-0.78251	2.98917	1.91854		
1.17500	0.03917	0.19D 00	-0.76230	2.67629	1.74785		
1.20000	0.04000	0.37D 00	-0.74120	2.41730	1.59688		
1.22500	0.04083	0.68D 00	-0.72046	2.20139	1.46478		
1.25000	0.04167	0.12D 01	-0.69999	2.02003	1.34971		

1.27500	0.04250	[illegible]	[illegible]	[illegible]	[illegible]
1.30000	0.04333	0.36D 01	-0.66033	1.73570	1.16240
1.32500	0.04417	0.69D 01	-0.64134	1.62346	1.08658
1.35000	0.04500	0.97D 01	-0.62300	1.52661	1.01997
1.37500	0.04583	0.15D 02	-0.60534	1.44262	0.96185
1.40000	0.04667	0.24D 02	-0.58841	1.36944	0.91089
1.42500	0.04750	0.37D 02	-0.57222	1.30546	0.86614
1.45000	0.04833	0.55D 02	-0.55679	1.24934	0.82680
1.47500	0.04917	0.80D 02	-0.54212	1.20003	0.79220
1.50000	0.05000	0.12D 03	-0.52823	1.15657	0.76174
1.52500	0.05083	0.16D 03	-0.51510	1.11830	0.73494
1.55000	0.05167	0.23D 03	-0.50274	1.08453	0.71135
1.57500	0.05250	0.31D 03	-0.49114	1.05490	0.69065
1.60000	0.05333	0.41D 03	-0.48026	1.02884	0.67246
1.62500	0.05417	0.54D 03	-0.47016	1.00603	0.65658
1.65000	0.05500	0.70D 03	-0.46077	0.98617	0.64271
1.67500	0.05583	0.89D 03	-0.45210	0.96900	0.63067
1.70000	0.05667	0.11D 04	-0.44412	0.95431	0.62027
1.72500	0.05750	0.14D 04	-0.43684	0.94190	0.61135
1.75000	0.05833	0.17D 04	-0.43024	0.93162	0.60377
1.77500	0.05917	0.20D 04	-0.42430	0.92335	0.59743
1.80000	0.06000	0.23D 04	-0.41903	0.91694	0.59221
1.82500	0.06083	0.26D 04	-0.41440	0.91234	0.58802
1.85000	0.06167	0.29D 04	-0.41040	0.90946	0.58479
1.87500	0.06250	0.32D 04	-0.40704	0.90823	0.58245
1.90000	0.06333	0.35D 04	-0.40429	0.90863	0.58095
1.92500	0.06417	0.37D 04	-0.40216	0.91060	0.58024
1.95000	0.06500	0.39D 04	-0.40065	0.91412	0.58029
1.97500	0.06583	0.40D 04	-0.39974	0.91919	0.58107
2.00000	0.06667	0.41D 04	-0.39944	0.92581	0.58256

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NUMBER OF HARMONICS CONSIDERED= 4

W= 1.000 XMC0=-0.4000 XF= 2.000 A2= 0.100 DD= 1.000
CR= 0.0 0.0 RI= 0.0050 NX= 80 NX1= 80 XMU= 0.0

X	X/L	DET	MACH NO.	FIRST HARMON.	SECOND HARMON.	THIRD HARMON.	FOURTH HARMON.
0.02500	0.00083	0.30D 07	-0.40030	1.00104	0.00051	0.00001	0.00000
0.05000	0.00167	0.28D 07	-0.40121	1.00416	0.00103	0.00001	0.00000
0.07500	0.00250	0.26D 07	-0.40273	1.00937	0.00155	0.00002	0.00000
0.10000	0.00333	0.23D 07	-0.40487	1.01673	0.00209	0.00002	0.00000
0.12500	0.00417	0.20D 07	-0.40762	1.02626	0.00265	0.00003	0.00000
0.15000	0.00500	0.17D 07	-0.41099	1.03804	0.00323	0.00004	0.00000
0.17500	0.00583	0.14D 07	-0.41499	1.05215	0.00384	0.00004	0.00000
0.20000	0.00667	0.11D 07	-0.41963	1.06867	0.00450	0.00005	0.00000
0.22500	0.00750	0.82D 06	-0.42492	1.08773	0.00520	0.00006	0.00000
0.25000	0.00833	0.60D 06	-0.43087	1.10945	0.00595	0.00008	0.00000
0.27500	0.00917	0.43D 06	-0.43749	1.13402	0.00677	0.00009	0.00000
0.30000	0.01000	0.30D 06	-0.44479	1.16163	0.00767	0.00011	0.00000
0.32500	0.01083	0.20D 06	-0.45278	1.19242	0.00867	0.00013	0.00000
0.35000	0.01167	0.13D 06	-0.46148	1.22673	0.00979	0.00016	0.00000
0.37500	0.01250	0.81D 05	-0.47090	1.26484	0.01104	0.00019	0.00000
0.40000	0.01333	0.49D 05	-0.48104	1.30710	0.01246	0.00023	0.00001
0.42500	0.01417	0.29D 05	-0.49193	1.35390	0.01408	0.00028	0.00001
0.45000	0.01500	0.17D 05	-0.50357	1.40573	0.01595	0.00034	0.00001
0.47500	0.01583	0.92D 04	-0.51597	1.46316	0.01813	0.00041	0.00001
0.50000	0.01667	0.49D 04	-0.52914	1.52684	0.02070	0.00051	0.00002
0.52500	0.01750	0.25D 04	-0.54309	1.59758	0.02373	0.00063	0.00002
0.55000	0.01833	0.13D 04	-0.55781	1.67633	0.02737	0.00080	0.00003
0.57500	0.01917	0.62D 03	-0.57333	1.76422	0.03173	0.00101	0.00004